<u>Reference: Reflections, shifts, stretches and compressions</u>
Mr. Chvatal

<u>Reflections</u>

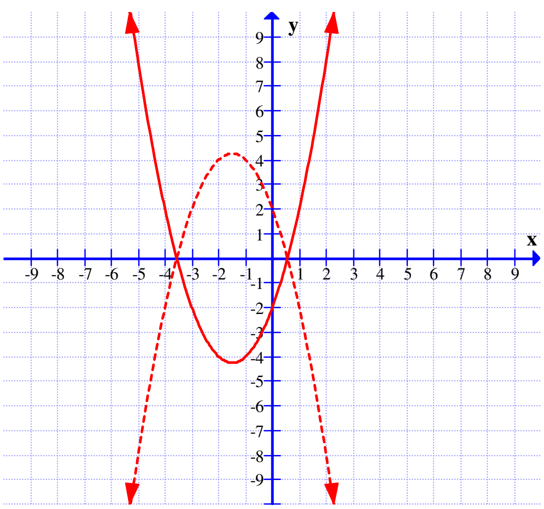
$y = -f(x)$	Reflection across x axis
$y = f(-x)$	Reflection across y axis
$y = -f(-x)$	Reflection across origin

If $f(x) = f(-x)$, the function is *even* and symmetrical about the y axis.

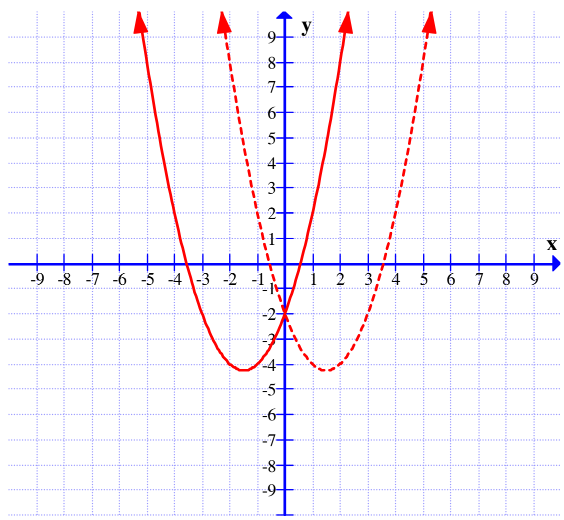
If $f(-x) = -f(x)$, the function is *odd* and symmetrical about the origin.

Example 1: The reflection of $f(x) = x^2 + 3x - 2$ over the x-axis is $g(x) = -x^2 - 3x + 2$, since $-f(x) = -x^2 - 3x + 2$.

Example 2: The reflection of $f(x) = x^2 + 3x - 2$ over the y-axis is $h(x) = x^2 - 3x - 2$, since $f(-x) = (-x)^2 + 3(-x) - 2 = x^2 - 3x - 2$.



Example 1



Example 2

Shifts

$y = f(x) + k$ Shift up by k units (outside change)

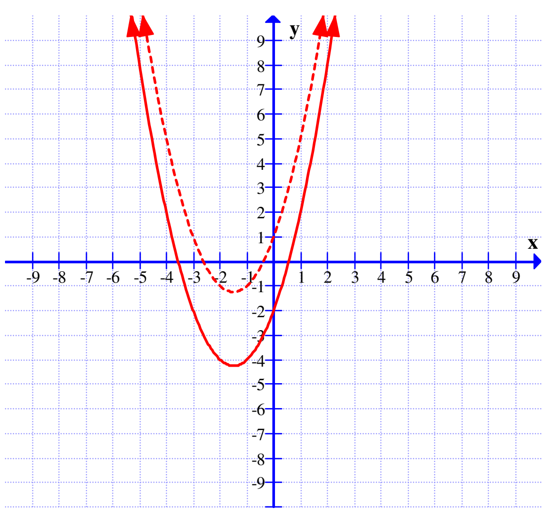
$y = f(x+k)$ Shift left by k units (inside change)

Example 3: The function $f(x) = x^2 + 3x - 2$ shifted up by 3 units is $g(x) = x^2 + 3x + 1$, since

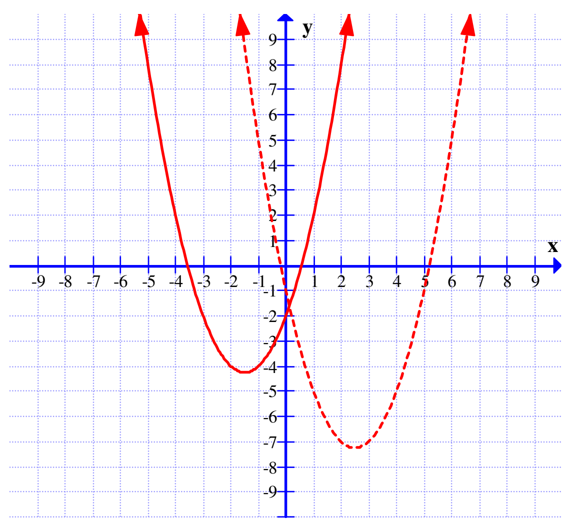
$$\begin{aligned} f(x) + 3 &= x^2 + 3x - 2 + 3 \\ &= x^2 + 3x + 1 \end{aligned}$$

Example 4: The function $f(x) = x^2 + 3x - 2$ shifted down by three units and right by four units is $h(x) = x^2 - 5x - 1$, since:

$$\begin{aligned} f(x-4) - 3 &= (x-4)^2 + 3(x-4) - 2 - 3 \\ &= \left(x^2 - 8x + 16\right) + (3x - 12) - 5 \\ &= x^2 - 5x - 1 \end{aligned}$$



Example 3



Example 4

Stretches and compressions

The function $y = k \bullet f(x)$ is $y = f(x)$:

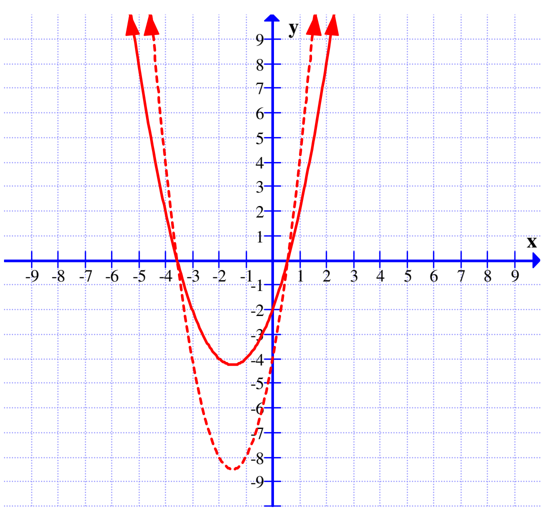
- Vertically stretched by k if $k > 1$
- Vertically compressed by k if $0 < k < 1$
- Either compressed or stretched by $|k|$ and reflected across the x axis if $k < 0$

The function $y = f\left(\frac{1}{k}x\right)$ is $y = f(x)$:

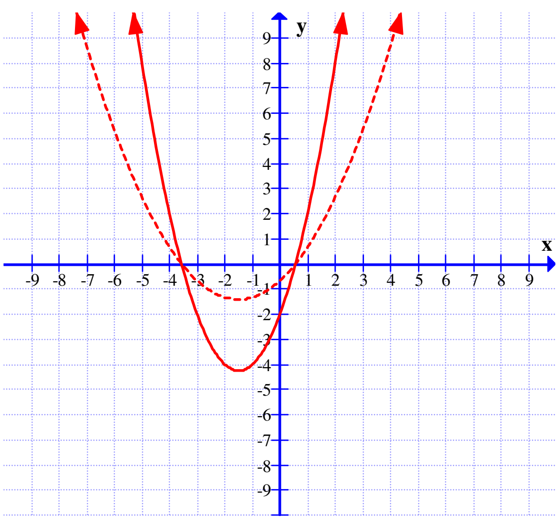
- Horizontally compressed by k if $0 < k < 1$
- Horizontally stretched by k if $k > 1$
- Either compressed or stretched by $|k|$ and reflected across the x axis if $k < 0$

Example 5: The function $f(x) = x^2 + 3x - 2$ vertically stretched by a factor of 2 is $g(x) = 2x^2 + 6x - 4$, since $2 \bullet f(x) = 2\left(x^2 + 3x - 2\right) = 2x^2 + 6x - 4$.

Example 6: The function $f(x) = x^2 + 3x - 2$ vertically compressed by a factor of $1/3$ is $h(x) = \frac{1}{3}x^2 + x - \frac{2}{3}$, since $\frac{1}{3} \bullet f(x) = \frac{1}{3}\left(x^2 + 3x - 2\right) = \frac{1}{3}x^2 + x - \frac{2}{3}$.



Example 5



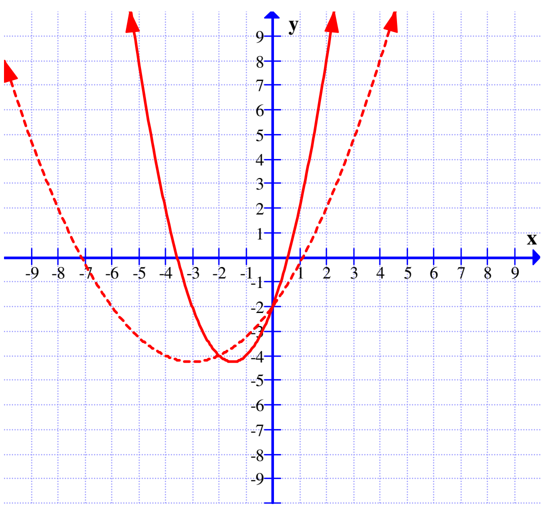
Example 6

Example 7: The function $f(x) = x^2 + 3x - 2$ horizontally stretched by a factor of 2 is $g(x) = \frac{1}{4}x^2 + \frac{3}{2}x - 2$, since:

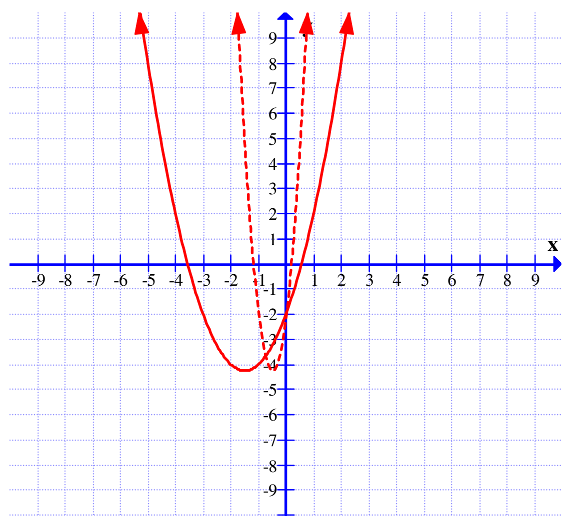
$$f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2 + 3\left(\frac{1}{2}x\right) - 2$$
$$= \frac{1}{4}x^2 + \frac{3}{2}x - 2$$

Example 8: The function $f(x) = x^2 + 3x - 2$ horizontally compressed by a factor of $\frac{1}{3}$ is $h(x) = 9x^2 + 9x - 2$, since:

$$f(3x) = (3x)^2 + 3(3x) - 2$$
$$= 9x^2 + 9x - 2$$



Example 7



Example 8