<u>Reference – absolute value transformations</u>
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<u>Vertex</u>

Given the function $f(x) = |x-h| + k$, the vertex is at the point (h,k) .

<u>Shifts</u>

$g(x) = f(x) + k$	Shift up by *k* units (outside change)
$h(x) = f(x+k)$	Shift left by *k* units (inside change)

Example 1
Let $f(x) = 2|x-1|$, write the rule for $k(x)$ if $k(x)$ is a shift two units down and three units to the right. Determine the vertex.

Answer

$k(x) = 2\|x-1(-3)\| - 2$	Subtracting on the inside shifts the graph 3 units to the right; subtracting two outside shifts it two units down.
$k(x) = 2\|x-4\| - 2$	The correct rule. The vertex is (4, -2).

<u>Reflections</u>

$g(x) = -f(x)$	Reflection across *x* axis
$h(x) = f(-x)$	Reflection across *y* axis
$k(x) = -f(-x)$	Reflection across origin

Example 2
Let $f(x) = 2|x-1|$, write the rule for $m(x)$ if $m(x)$ is a reflection across the *y*-axis. Determine the vertex.

Answer

$m(x) = 2\|(-x)+1)\|$	Replacing *x* with -*x* reflects the graph across the *y*-axis.
$m(x) = 2\|-x+1\|$	The correct rule. The vertex is (-1, 0).

Stretches and compressions

The function $g(x) = k \bullet f(x)$ is $f(x)$:

- Vertically stretched by k if $k > 1$
- Vertically compressed by k if $0 < k < 1$
- Either compressed or stretched by $|k|$ and reflected across the x axis if $k < 0$

The function $g(x) = f(kx)$ is $f(x)$:

- Horizontally compressed by k if $k > 1$, or by $\frac{1}{k}$ if $k < 1$
- Horizontally stretched by k if $k < 1$, or by $\frac{1}{k}$ if $k > 1$

Example 3
Stretch the graph of $f(x) = |x| - 2$ vertically by a factor of 3 and determine the vertex.

Answer

$g(x) = 3(\lvert x \rvert - 2)$	Multiply $f(x)$ by 3.
$g(x) = 3\lvert x \rvert - 6$	The correct rule. The vertex is (0,-6).

Example 4
Compress the graph of $f(x) = |x-1| - 3$ horizontally by a factor of 0.5 and determine the vertex.

Answer

$g(x) = f\left(\frac{1}{k}x\right)$	
$g(x) = \left\lvert \frac{1}{0.5}x - 1 \right\rvert - 3$	Multiply x by $\frac{1}{k}$, or $\frac{1}{0.5}$.
$g(x) = \lvert 2x - 1 \rvert - 3$	The correct rule. The vertex is $\left(\frac{1}{2}, -3\right)$.