<u>**Notes – A review of linear functions**</u>

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The table below shows the relationship between the cost of driving a rental car (*C*) and distance (*d*).

C	*d*
$21	4
$24	16
$26.75	27
$27.25	29
$31	44

Does this table describe a relation?

Yes. It is a set of ordered pairs.

Is *C* a function of *d*?

Yes. Each value of *d* maps to a unique value of *C*. You can also test by graphing the equation that describes this function, and applying the vertical line test.

Is *d* a function of *C*?

Yes. Each value of *C* maps to a unique value of *d*. You can also test by graphing the equation that describes this function, and applying the horizontal line test.

Which is the independent variable and which is the dependent?

The rental cost (*C*) of the car *depends on* how far (*d*) it is driven. Therefore *C* is the dependent variable and *d* is the independent. The dependent variable is represented by *y*, and graphed on the *y* axis, and the independent is represented by *x* and graphed on the *x* axis. We can also say that, in real terms, *C* is a function of *d*.

How can we determine if this relationship is in fact linear?

We can graph it, of course, but even then it's hard to tell if the graph of the data is a straight line. But since we know that all linear relations are characterized by a *constant rate of change*, we can check the rate of change for each interval to make our determination. We can calculate the rate of change by dividing the change in *C* by the change in *d*:

$$\text{Rate of change} = \frac{\Delta C}{\Delta d}, \text{ or } \frac{C_1 - C_0}{d_1 - d_0}$$

Let's try the first interval:

$$\text{Rate of change} = \frac{C_1 - C_0}{d_1 - d_0} = \frac{24 - 21}{16 - 4} = \frac{3}{12} = .25$$

Now the second interval:

$$\text{Rate of change} = \frac{C_1 - C_0}{d_1 - d_0} = \frac{26.75 - 24}{27 - 16} = \frac{2.25}{9} = .25$$

Now, just for an experiment, the interval between the third and final data point:

$$\text{Rate of change} = \frac{C_1 - C_0}{d_1 - d_0} = \frac{31 - 26.75}{44 - 27} = \frac{4.25}{17} = .25$$

In fact, the rate of change is a constant .25 no matter which interval we choose. Therefore, the relationship is linear.

One last point: it is also possible to determine if a relationship is linear by using the linear regression function on your calculator. More on that later.

What is the rate of change?

As we determined above, it is .25, or $0.25 per mile. Note that rate of change is the same as *slope*, represented by *m* in the equation $y = mx + b$.

Can you write the linear equation that describes the relation?

What we're looking for here is an equation of the form $y = mx + b$, or in this case $C = md + b$. Now that we have the rate of change (*m*), we can plug in any value for (*d*, *C*) from our table and find the intercept. I've chose the data point (16, 24).

$C = .25d + b$

$24 = .25(16) + b$	Evaluate for $C = 24$ and $d = 16$.
$24 = 4 + b$	Solve for *b*.
$20 = b$	The *y*-intercept is 20.

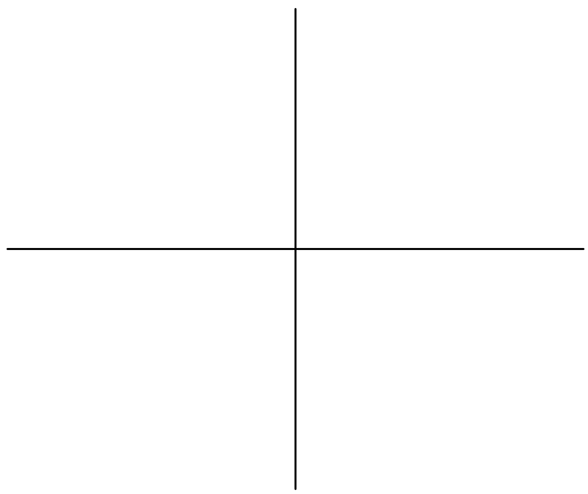
Therefore, the equation is $C = .25d + 20$.

Can you write this equation in functional notation?

Since C is a function of d, we can substitute $f(d)$ for C and write: $f(d) = .25d + 20$.

Can you graph the relation on the coordinate plane?

Since C is the dependent variable, it is graphed along the y axis, and d along the x axis.



Can you find the x and y intercepts algebraically?

Yes, by setting $x = 0$ for the y-intercept, and $y = 0$ for the x-intercept. We'll first write $C = .25d + 20$ in $y = mx + b$ form, getting $y = .25x + 20$. Then, to find the y-intercept:

$y = .25(0) + 20$	Evaluate for $x = 0$.
$y = 20$	The y-intercept is 20.

And to find the x-intercept:

$0 = .25x + 20$	Evaluate for $y = 0$
$-.25x = 20$	
$x = -80$	The x-intercept is -80.

In this instance, what does the *y*-intercept mean?

It means it costs $20 to rent the car prior to a mileage charge.

In this instance, what does the *x*-intercept mean?

Literally it means that for the rental cost of the car to be $0, you'd have to drive negative 80 miles. It's not a meaningful calculation.

Can you find the inverse of the relation?

An inverse of a relation is an *exchange* of the two variables, such as *x* and *y*. In this case, let's begin by exchanging *d* and *C*, and then solve for *C*.

$C = .25d + 20$

$d = .25C + 20$ Exchange *C* and *d*.

$.25C = d - 20$ Subtract 20 from each side.

$C = 4d - 80$ Multiply each side by 4.

The inverse is $C = 4d - 80$, or $f(d) = 4d - 80$.

Is the inverse a function? Why or why not?

Yes. Each value of *d* maps to a unique value of *C*. The graph of this relation also passes the vertical line test.

How much would it cost to drive 65 miles?

This is an example of *extrapolation*, which is predicting a result *outside of* the data you are given. We can extrapolate by substituting 65 for *d* and solving for *C*. In this case I'll use functional notation, and replace *C* with $f(d)$.

$f(d) = .25d + 20$

$f(65) = .25(65) + 20$ Evaluate for $d = 65$

$f(65) = 36.25$ Solve.

It would cost $36.25 to rent the car and drive 65 miles.

How much would it cost to drive 19 miles?

This is an example of *interpolation*, which is predicting a result *between* the data points you are given. We can interpolate by substituting 19 for d and solving for C.

$f(d) = .25d + 20$

$f(19) = .25(19) + 20$ Evaluate for $d = 19$

$f(65) = 36.25$ Solve.

It would cost \$36.25 to rent the car and drive 65 miles.

If your cost is \$60, how far did you drive?

Here C, or in functional notation $f(d)$, is known and we need to determine d. But the process is the same.

$f(d) = .25d + 20$

$60 = .25d + 20$ Evaluate for $f(d) = 60$

$40 = .25d$ Solve for d.

$160 = d$

If your final cost is \$60, you drove 160 miles.